

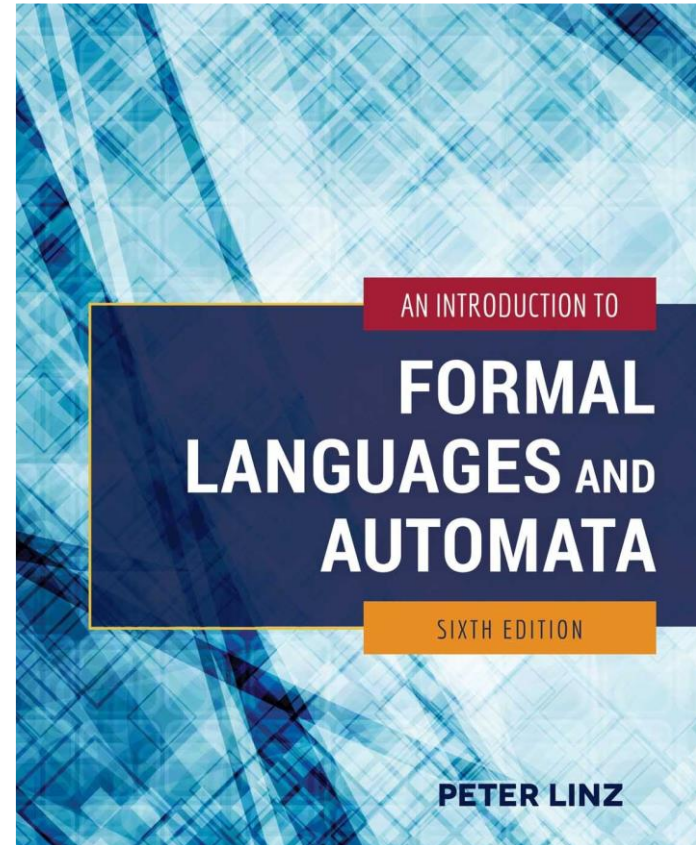
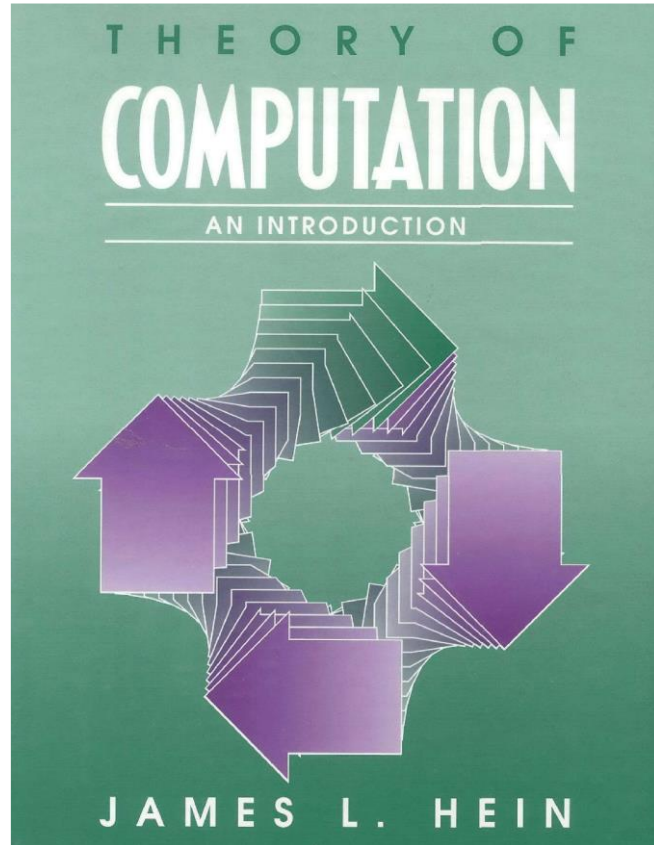
# Automata and Formal Languages

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Lecture 08

# Books

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# PowerPoint

<http://www.bu.edu.eg/staff/ahmedaboalatah14-courses/14767>

The screenshot shows a web interface for Benha University. At the top, there is a blue header with the university logo, the name 'Benha University', and a welcome message for 'Ahmed Hassan Ahmed Abu El Atta' with a 'Log out' link. Below the header, a navigation menu on the left lists various university services. The main content area displays course details for 'Automata and Formal Languages' by 'Ass. Lect. Ahmed Hassan Ahmed Abu El Atta'. The details are presented in a table with blue headers and white content. A 'Course password' section is also visible. On the right side, there are social media icons and a vertical toolbar with icons for Google, a book, RG, LinkedIn, Facebook, Twitter, Google+, YouTube, WordPress, a camera, a globe, a question mark, and an edit icon.

Benha University

Staff Search: **Welcome: Ahmed Hassan Ahmed Abu El Atta (Log out)**

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**Ass. Lect. Ahmed Hassan Ahmed Abu El Atta :: Course Details:**  
**Automata And Formal Languages** [add course](#) | [edit course](#)

Course name	Automata and Formal Languages
Level	Undergraduate
Last year taught	2018
Course description	Not Uploaded
Course password	
Course files	<a href="#">add files</a>
Course URLs	<a href="#">add URLs</a>
Course assignments	<a href="#">add assignments</a>
Course Exams & Model Answers	<a href="#">add exams</a>

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# Minimum-State DFAs

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MINIMUM-STATE DFAS

# Agenda

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- Minimum-State DFAs
- Example 1
- Minimum-State DFAs Algorithm
- Example 2
- Example 3

# Minimum-State DFAs

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One way to try and simplify the DFA for some regular expression is to algebraically transform the regular expression into a simpler one before starting construction of the DFA.

$$\lambda + a + aaa^* = a^*.$$

\*Every **regular expression** has a **unique** minimum-state DFA.

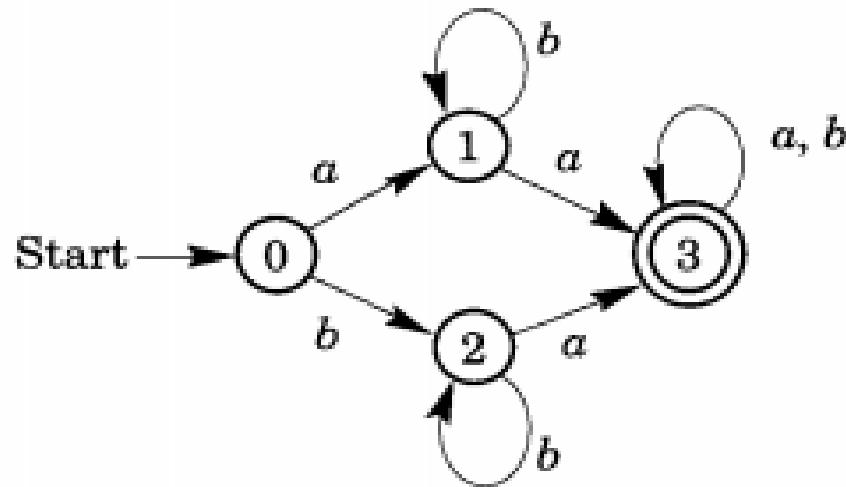
# How to transform a DFA into a minimum state DFA?

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\*The key idea is to **define** two states **s** and **t** to be **equivalent** if for every string **w**, the transitions

$T(s, w)$  and  $T(t, w)$  are either both **final** or both **nonfinal**.

# Example 1:



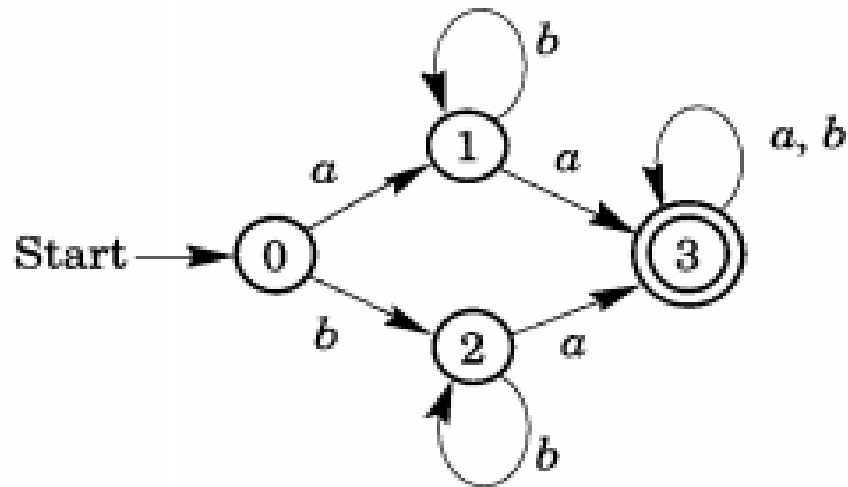
	<i>T</i>	<i>a</i>	<i>b</i>
start	0	1	2
	1	3	1
	2	3	2
final	3	3	3

States 1 and 2 are equivalent. So for any string  $w$ , both  $T(1, w)$  and  $T(2, w)$  are either both final or both nonfinal.

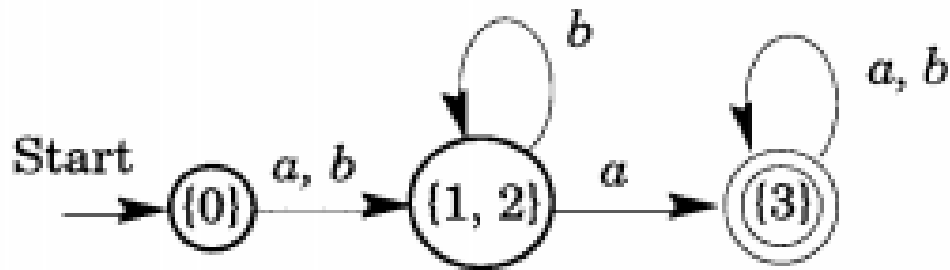
$\{0\}$ ,  $\{1, 2\}$ , and  $\{3\}$ .



# Example 1:



	$T$	$a$	$b$
start	0	1	2
	1	3	1
final	2	3	2
	3	3	3



	$T_{\min}$	$a$	$b$
start	{0}	{1, 2}	{1, 2}
	{1, 2}	{3}	{1, 2}
final	{3}	{3}	{3}

# Minimum-State DFAs

**Input**: A DFA with set of states  $S$  and transition table  $T$ . Assume

**Output**: A minimum-state DFA recognizing the same regular language as the input DFA.

1. Construct the equivalent pairs of states by calculating the descending sequence of sets of pairs  $E_0 \supset E_1 \supset \dots$  defined as follows:

2.  $E_0 = \{(s, t) \mid s \text{ and } t \text{ are distinct and either both states are final or both states are non-final}\}$ .

$$E_{i+1} = \{(s, t) \mid (s, t) \in E_i \text{ and for every } a \in A \text{ either } T(s, a) = T(t, a) \text{ or } \{T(s, a), T(t, a)\} \in E_i\}.$$

The computation stops when  $E_k = E_{k+1}$  for some index  $k$ .

3. The start state is the equivalence class containing the start state.

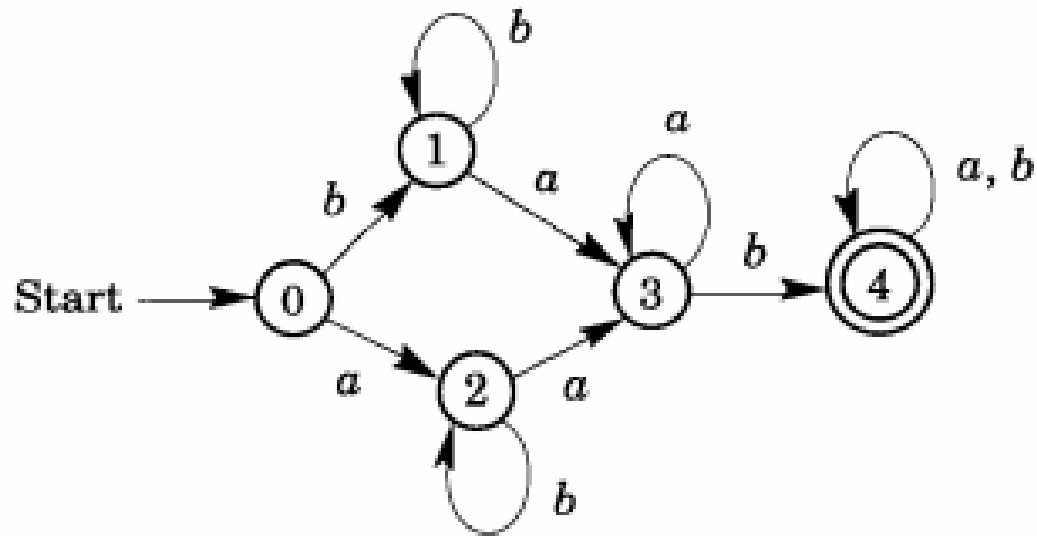
4. A final state is any equivalence class containing a final state .

5. The transition table  $T_{\min}$  for the minimum-state DFA is defined as follows, where  $[s]$  denotes the equivalence class containing  $s$  and  $a$  is any letter:  $T_{\min}([s], a) = [T(s, a)]$ .

*End of Algorithm*

# Example 2

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T	a	b
0	2	1
1	3	1
2	3	2
3	3	4
4	4	4

T	a	b
0	2	1
1	3	1
2	3	2
3	3	4
4	4	4

## Example 2

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$$E_0 = \{\{0, 1\}, \{0, 2\}, \{0, 3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}.$$

$$\{T(s, x), T(t, x)\} \in E_0 \text{ or } T(s, x) = T(t, x).$$

$$\{T(0, a), T(1, a)\} = \{2, 3\} \in E_0.$$

$$T(0, b) = T(1, b).$$

$$\{T(0, a), T(3, a)\} = \{2, 3\} \in E_0.$$

$$T(0, b) = \{1\}$$

$$T(3, b) = \{4\}. \text{ Eliminate } \{0, 3\}$$

$$E_1 = \{\{0, 1\}, \{0, 2\}, \{1, 2\}\}.$$

$$E_2 = \{\{1, 2\}\}, \quad E_3 = E_2 = \{\{1, 2\}\}.$$

# Example 2

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$\{0\}, \{1, 2\}, \{3\}, \{4\}$

Tmin	a	b
<b>{0}</b>	{1,2}	{1,2}
{1,2}	{3}	{1,2}
{3}	{3}	<b>{4}</b>
<b>{4}</b>	<b>{4}</b>	<b>{4}</b>

# Example 3

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compute the minimum-state DFA for the DFA given by the following transition table:

T	a	b
0	1	2
1	4	1
2	4	3
3	4	3
4	4	5
5	5	5

# Example 3

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$$E_0 = \{\{0, 1\}, \{0, 2\}, \{0, 3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{4, 5\}\}.$$

$$E_1 = \{\{1, 2\}, \{1, 3\}, \{2, 3\}, \{4, 5\}\}.$$

$$E_2 = E_1 = \{\{1, 2\}, \{1, 3\}, \{2, 3\}, \{4, 5\}\}.$$

Three classes  $\{0\}, \{1, 2, 3\}, \{4, 5\}$

T	a	b
0	1	2
1	4	1
2	4	3
3	4	3
4	4	5
5	5	5

# Example 3

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Three classes  $\{0\}$ ,  $\{1,2,3\}$ ,  $\{4,5\}$

T	a	b
$\{0\}$	$\{1,2,3\}$	$\{1,2,3\}$
$\{1,2,3\}$	$\{4,5\}$	$\{1,2,3\}$
$\{4,5\}$	$\{4,5\}$	$\{4,5\}$



